Magneto-Acoustic Waves of Small Amplitude in Optically Thin Quasi-Isentropic Plasmas

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ABSTRACT

The evolution of quasi-isentropic magnetohydrodynamic waves of small but finite amplitude in an optically thin plasma is analyzed. The plasma is assumed to be initially homogeneous, in thermal equilibrium and with a straight and homogeneous magnetic field frozen in. Depending on the particular form of the heating/cooling function, the plasma may act as a dissipative or active medium for magnetoacoustic waves, while Alfvén waves are not directly affected. An evolutionary equation for fast and slow magnetoacoustic waves in the single wave limit, has been derived and solved, allowing us to analyse the wave modification by competition of weakly nonlinear and quasi-isentropic effects. It was shown that the sign of the quasi-isentropic term determines the scenario of the evolution, either dissipative or active. In the dissipative case, when the plasma is first order isentropically stable the magnetoacoustic waves are damped and the time for shock wave formation is delayed. However, in the active case when the plasma is isentropically overstable, the wave amplitude grows, the strength of the shock increases and the breaking time decreases. The magnitude of the above effects depends upon the angle between the wave vector and the magnetic field. For hot ($T > 10^4$ K) atomic plasmas with solar abundances either in the interstellar medium or in the solar atmosphere, as well as for the cold ($T < 10^3$ K) ISM molecular gas, the range of temperature where the plasma is isentropically unstable and the corresponding time and length-scale for wave breaking have been found.

Subject headings: Magnetoacoustic Waves, Thermal Instability, Shock Waves, Interstellar Medium, Solar Atmosphere

1. Introduction

The study of wave propagation and the stability of optically thin plasmas is of crucial importance for understanding the origin and evolution of inhomogeneities observed at very different length scales and time scales in astrophysical plasmas. Travelling acoustic waves in the plasma can be unstable due to peculiarities of the processes of heating and cooling of the medium. The possible amplification of the waves is connected with additional energy liberation in the plasma in the compression phase or, in other words, with nonadiabaticity of the plasma. In different astrophysical applications, the nonadiabaticity of the plasma can be mathematically modelled by an introduction of an additional term in the adiabatic equation. This term, usually called a heating/cooling function, depends on parameters of the plasma under consideration. The specific expression for this function varies in different physical situation (Rosner, et. al., 1978; Vesecky et. al., 1979; Dahlburg and Mariska, 1988), but fortunately some general properties of this function define main features of dynamics of perturbations in optically thin plasmas (either amplification or decay).

The above problems, taking into account many different physical effects have been extensively studied as far as the linear approximation is concerned (Parker, 1953; Kruskal and Schwarzschild, 1966; Weymann, 1960; Field, 1965; Priest, 1982; Ibáñez and Parravano, 1983; Ibáñez, 1985; Ibaáñez and Escalona, 1993; Bodo et al.,1985; Balbus and Soker, 1989 and references therein). The linear approach corresponds to the case of waves with zero amplitude and, consequently, can describe only an initial stage of the instabilities associated with nonadiabatic effects in the plasma. Generally, the nonlinear regime is not far well understood and most of the progress has been done by numerical simulations of very particular structures (Dahlburg et al., 1987; Karpen et al., 1989; Reale et al., 1994).

In addition to condensations that can be formed in hot plasmas by thermal instability in a nonadiabatic plasma, providing that the instability does not saturate in the nonlinear regime, the steeping of travelling waves may also originate inhomogeneities (Oppenheimer, 1977; Krasnobaev, 1975; Krasnobaev and Tarev, 1987 (Reference KT herein after); Tarev, 1993). This problem has been worked out in KT and Tarev (1993) references neglecting the presence of magnetic fields. However, in many astrophysical situations the magnetic fields become relevant, therefore, it is worthy to analyse the propagation of nonlinear magnetoacoustic waves in plasmas where nonadiabatic effects also go into play. This problem beyond its heuristic interest, also can be the clue for understanding particular observed features in magnetised plasmas, for instance, the intensity oscillations observed in the network bright points in the quiet Sun (Kalkofen, 1997). Also, in the presence of the magnetic field, the magnetoacoustic waves perturb the absolute value and components of the field, which gives an additional possibility for the registration of the discussed phenomena in astrophysical plasmas through observation of variation of gyro-emission, Zeeman effect etc.

The aim of the present paper is the general study of weakly nonlinear magnetoacoustic and Alfvén wave dynamics in presence of nonadiabatic effects in an optically thin plasma. We consider a homogeneous plasma penetrated by the straight and homogeneous magnetic field. Effects of self-gravitation, ionization and steady flows are neglected. The physical state is determined by time and spatial independent variables. Additionally, the heating/cooling function (per unit volume and time) is assumed to be dependent of pressure and density, i.e. $Q(p, \rho)$. Only quadratic nonlinear effects will be taken into account. Assumption that nonlinear and nonadiabatic effects are weak, but not necessarily of the same order, allows us to apply the method of slowly varying amplitudes and derive an evolu-

tionary equation for fast and slow magnetoacoustic waves. Note that, despite the nonlinearity considered in the paper is weak, it leads to the significant modification of the wave, up to formation of shocks. In the derivation of the evolutionary equation, a single wave approximation is applied. This means that interaction of waves of different types, as well as interaction of forward and backward waves, is neglected. The nonlinear interaction of different modes, for example, three-wave interaction, occurs when all interacting modes are existing in the system (and are, actually, over some certain amplitude threshold prescribed by dissipation). Also, efficiency of the interaction depends strongly on coherence of the interacting waves. Both these conditions are assumed to be not fulfilled: we consider a wave of one certain type only, while other modes are not excited. Also, our attention is restricted to evolution of wave pulses with a wide spectrum, and nonlinear resonant interactions are suppressed. Thus, the dominating nonlinear process is generation of higher harmonics, leading to wave steepening and shock formation. Moreover, this process is more pronounced due to the absence of linear high frequency dispersion. Nonlinear dispersion preventing the second harmonics generation, may appear in the next order of nonlinearity and is out of scopes of the paper.

Since the thermally unstable plasma is an *active* medium for the magnetoacoustic waves, the solution of the evolutionary equation shows the amplification of the initial magnetoacoustic perturbation and speeding up of shock formation. The breaking time depends on the angle of the wave propagation with respect to the magnetic field and is different for slow and fast waves. The above results are quite general and potentially can be applied to different astrophysical as well as laboratory plasmas. However, in this paper, the results obtained will be addressed to study a hot ($T > 10^4$ K) atomic plasma (Solar atmosphere or hot Interstellar Medium) and a cold ($T < 10^3$ K) molecular gas (say for instance, the cold ISM gas).

2. Governing equations

In the model considered, dynamics of the plasma is described by the set of MHD equations,

$$\frac{\partial \mathbf{B}}{\partial t} = \text{curl } \mathbf{V} \times \mathbf{B}, \tag{1}$$

$$\rho \frac{d\mathbf{V}}{dt} = -\nabla p - \frac{1}{4\pi} \mathbf{B} \times \text{curl } \mathbf{B}, \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \text{div } \rho \mathbf{V} = 0, \tag{3}$$

$$\frac{dp}{dt} - \frac{\gamma p}{\rho} \frac{d\rho}{dt} = (\gamma - 1) \ Q(p, \rho), \tag{4}$$

$$\text{div} \mathbf{B} = 0, \tag{5}$$

where $Q(p, \rho)$ is a known function describing effects of the nonadiabaticity and all other notations are standard.

The stationary magnetic field is in the $xz-$plane, $\mathbf{B_0} = B_0 \sin\alpha \ \mathbf{x_0} + B_0 \cos\alpha \mathbf{z_0}$, where B_0 is the absolute value of the magnetic field, α is the angle between the magnetic field and $z-$axis, $\mathbf{x_0}$ and $\mathbf{z_0}$ are the unit vectors. We consider dynamics of waves propagating along the $z-$axis. Dependences upon x and y are ignored ($\partial/\partial x = \partial/\partial y = 0$).

Projecting Eqs. (1)—(4) on the axes, we have

$$\frac{\partial B_x}{\partial t} = -\frac{\partial}{\partial z}(V_z B_x - V_x B_z), \tag{6}$$

$$\frac{\partial B_y}{\partial t} = \frac{\partial}{\partial z}(V_y B_z - V_z B_y), \tag{7}$$

$$\frac{\partial B_z}{\partial t} = 0, \tag{8}$$

$$\rho \frac{d \ V_x}{d \ t} = \frac{1}{4\pi} B_z \frac{\partial B_x}{\partial z}, \tag{9}$$

$$\rho \frac{d\,V_y}{d\,t} = \frac{1}{4\pi} B_z \frac{\partial B_y}{\partial z}, \tag{10}$$

$$\rho \frac{d\,V_z}{d\,t} = -\frac{\partial p}{\partial z} - \frac{1}{4\pi}\left(B_x \frac{\partial B_x}{\partial z} + B_y \frac{\partial B_y}{\partial z}\right), \tag{11}$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z}\left(\rho\, V_z\right), \tag{12}$$

$$\frac{\partial p}{\partial t} + V_z \frac{\partial p}{\partial z} - \frac{\gamma p}{\rho}\left(\frac{\partial \rho}{\partial t} + V_z \frac{\partial \rho}{\partial z}\right) = (\gamma - 1) Q(p, \rho). \tag{13}$$

We rewrite equations (6)-(13) gathering linear and nonlinear terms on left and right handside, respectively,

$$\frac{\partial B_x}{\partial t} + \frac{\partial}{\partial z}\left(B_{0x} V_z - B_{0z} V_x\right) = N_1, \tag{14}$$

$$\frac{\partial B_y}{\partial t} - \frac{\partial}{\partial z}\left(B_{0z} V_y\right) = N_2, \tag{15}$$

$$\rho_0 \frac{\partial V_x}{\partial t} - \frac{B_{0z}}{4\pi}\frac{\partial B_x}{\partial z} = N_4, \tag{16}$$

$$\rho_0 \frac{\partial V_y}{\partial t} - \frac{B_{0z}}{4\pi}\frac{\partial B_y}{\partial z} = N_5, \tag{17}$$

$$\rho_0 \frac{\partial V_z}{\partial t} + \frac{\partial p}{\partial z} + \frac{B_{0x}}{4\pi}\frac{\partial B_x}{\partial z} = N_6, \tag{18}$$

$$\rho_0 \frac{\partial p}{\partial t} - \gamma p_0 \frac{\partial \rho}{\partial t} - \rho_0(\gamma - 1)\left(\frac{\partial Q}{\partial \rho}\rho + \frac{\partial Q}{\partial p} p\right) = N_7, \tag{19}$$

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial V_z}{\partial z} = N_8, \tag{20}$$

where $B_{0x} = B_0 \ \sin\alpha$ and $B_{0z} = B_0 \ \cos\alpha$; N_1, N_2, N_4–N_8 are nonlinear terms. The derivatives of the heating/cooling function Q in equation (19) are calculated for unperturbed values of the density and pressure, $\rho = \rho_0$ and $p = p_0$. According to equation (8), we choose the perturbation $B_z = 0$.

Restricting ourselves to keep quadratic nonlinear terms only, we obtain the following expressions for the nonlinear right handsides,

$$N_1 = -\frac{\partial}{\partial z}(V_z B_x), \tag{21}$$

$$N_2 = -\frac{\partial}{\partial z}\left(V_z B_y\right), \tag{22}$$

$$N_4 = -\rho \frac{\partial V_x}{\partial t} - \rho_0 V_z \frac{\partial V_x}{\partial z}, \tag{23}$$

$$N_5 = -\rho \frac{\partial V_y}{\partial t} - \rho_0 V_z \frac{\partial V_y}{\partial z}, \tag{24}$$

$$N_6 = -\rho \frac{\partial V_z}{\partial t} - \rho_0 V_z \frac{\partial V_z}{\partial z} - \frac{1}{4\pi} B_x \frac{\partial B_x}{\partial z} - \frac{1}{4\pi} B_y \frac{\partial B_y}{\partial z}, \tag{25}$$

$$N_7 = -\rho \frac{\partial p}{\partial t} - \rho_0 V_z \frac{\partial p}{\partial z} + \gamma p \frac{\partial \rho}{\partial t} + \gamma p_0 V_z \frac{\partial \rho}{\partial z}, \tag{26}$$

$$N_8 = -\frac{\partial}{\partial z}(\rho V_z). \tag{27}$$

In Eqs. (21)—(27), variables ρ, p, V_x and B_x may be expressed through the variable V_z from the following relations:

$$\frac{\partial \rho}{\partial t} = -\rho_0 \frac{\partial V_z}{\partial z}, \tag{28}$$

$$\frac{\partial p}{\partial t} = -\rho_0 C_s^2 \frac{\partial V_z}{\partial z}, \tag{29}$$

$$\frac{\partial^2 V_x}{\partial t^2} = -\cot\alpha \left(\frac{\partial^2}{\partial t^2} - C_s^2 \frac{\partial^2}{\partial z^2}\right)\ V_z, \tag{30}$$

$$\left(\frac{\partial^2}{\partial t^2} - C_A^2 \cos^2\alpha \frac{\partial^2}{\partial z^2}\right)\ B_x = -B_0 \sin\alpha \frac{\partial^2 V_z}{\partial z \partial t}. \tag{31}$$

Eqs. (15) and (17) can be combine to the equation for the Alfvén wave:

$$\mathcal{D}_{Az}\ V_y = \frac{1}{\rho_0}\left(\frac{\partial N_5}{\partial t} + \frac{B_{0z}}{4\pi}\frac{\partial N_2}{\partial z}\right), \qquad (32)$$

where we use the Alfvén wave operator,

$$\mathcal{D}_{Az} = \frac{\partial^2}{\partial t^2} - C_{Az}^2\frac{\partial^2}{\partial z^2}$$

and $C_{Az} = B_{0z}/(4\pi\rho_0)^{1/2}$.

Eqs. (14)-(20) can be combined to the equation for magnetacoustic waves:

$$\mathcal{D}_{Az}\mathcal{D}_s\ V_z - C_{Ax}^2\frac{\partial^4 V_z}{\partial z^2\partial t^2} - (\gamma-1)$$

$$\left(\frac{\partial Q}{\partial \rho} + C_s^2\frac{\partial Q}{\partial p}\right)\mathcal{D}_{Az}\int\frac{\partial^2 V_z}{\partial z^2}\ dt =$$

$$= \frac{1}{\rho_0}\left\{\mathcal{D}_{Az}\left[\frac{\partial N_6}{\partial t} - \frac{\partial}{\partial z}\left(C_s^2 N_8 + \frac{1}{\rho_0}N_7\right)\right]\right.$$

$$\left. -\frac{B_{0x}}{4\pi}\frac{\partial^2}{\partial z\partial t}\left(\frac{\partial N_1}{\partial t} + \frac{B_{0z}}{\rho_0}\frac{\partial N_4}{\partial z}\right)\right\}, \qquad (33)$$

where

$$\mathcal{D}_s = \frac{\partial^2}{\partial t^2} - C_s^2\frac{\partial^2}{\partial z^2},$$

$C_s = \sqrt{\gamma p_0/\rho_0}$ is the sound speed and $C_{Ax} = B_{0x}/(4\pi\rho_0)^{1/2}$.

In the linear limit, the Alfvén wave perturbs the variables B_y and V_y, the magnetosonic waves perturb the variables B_x, V_x, V_z, ρ and p. As it follows from (32), the introduction of the heating/cooling function $Q(p,\rho)$ does not change the propagation of the Alfvén waves, at least till the quadratic nonlinear terms.

Equations (32) and (33) form the governing set of equations for consideration of quadratically nonlinear MHD wave dynamics in an homogeneous plasma with nonadiabaticity. Set (32), (33) contains information about independent dynamics of the MHD waves (Alfvén and fast and slow magnetoacoustic) as well as their nonlinear interaction. This set is an analogue of the similar set of equations derived in by Nakariakov and Oraevsky (1995), Nakariakov et al. (1997a,b, 1998) for MHD waves of finite amplitude in smoothly inhomogeneous cold plasma.

An analysis of the right handsides of equations (32) and (33) shows that if the Alfvénic perturbations (i.e. B_y and V_y) are initially absent from the system, they can not be excited by the magnetoacoustic perturbations, because all terms on the right handside of (32) are *products* of Alfvén and magnetoacoustic variables. This gives an advantage for an analytical description of the nonlinear dynamics of the magnetoacoustic waves, allowing to take V_y and B_y zero always and everywhere. Strictly speaking, this assumption is non-physical, because Alfvénic perturbations are always presented in the realistic plasma as noise. Consequently, in the natural conditions there is a nonlinear interaction of Alfvén and magnetoacoustic waves. Nevertheless, if the level of Alfvénic noise is initially sufficiently low, the assumption can be applied to the description of magnetoacoustic waves in some certain initial stage of the interaction, when nonlinear terms including Alfvén variables are much less than magnetoacoustic terms.

3. Dispersion relations

Supposing $V_z \sim \exp(i\omega t - ikz)$, we obtain the dispersion relation for the magnetosonic waves from Eq. (33):

$$(\omega^2 - C_A^2\cos^2\alpha\ k^2)(\omega^2 - C_s^2 k^2) - C_A^2\sin^2\alpha\omega^2 k^2$$
$$+\frac{i}{\omega}Ak^2(\omega^2 - C_A^2\cos^2\alpha\ k^2) = 0, \qquad (34)$$

where $C_A = B_0/(4\pi\rho_0)^{1/2}$ and

$$A = (\gamma-1)\left(\frac{\partial Q}{\partial\rho} + C_s^2\frac{\partial Q}{\partial p}\right). \qquad (35)$$

If the heating/cooling function is not taken into account, $A = 0$, we have the well-known dispersion relation for magnetosonic waves,

$$(\omega^2 - C_A^2\cos^2\alpha\, k^2)(\omega^2 - C_s^2 k^2) - C_A^2\sin^2\alpha\,\omega^2 k^2 = 0, \qquad (36)$$

which describes dispersion for both fast and slow magnetosonic waves. Dispersion relation (36) gives that the phase speed is

$$C^2_{\text{fast, slow}}\frac{1}{2}\left[C_s^2+C_A^2\pm\sqrt{(C_s^2+C_A^2)^2-4\cos^2\alpha\; C_s^2C_A^2}\right], \tag{37}$$

where the upper sign corresponds to the fast wave and the lower to the slow wave. Figure (1) shows the variation of this speed as a function of α for different values of plasma beta (β).

Consider the case of the parallel propagation, $\alpha = 0$,

$$(\omega^2 - C_A^2k^2)(\omega^2 - C_s^2k^2 + \frac{i}{\omega}Ak^2) = 0. \tag{38}$$

There are fast and slow waves. The thermal instability can be only on the slow mode, in this case. If $C_A = 0$, we obtain the same result as in KT reference. If $C_s \to 0$ (the coronal case), we have $\omega^3 = -iAk^2$.

Consider transversal propagation, $\alpha = \pi/2$,

$$\omega^2\left(\omega^2 - (C_A^2 + C_s^2)k^2 + \frac{iA}{\omega}k^2\right) = 0, \tag{39}$$

there is not slow wave now, but the fast wave can be a subject to the thermal instability.

In the hydrodynamical case, $C_A = 0$, the dispersion equation becomes

$$\omega^2 - C_s^2k^2 + \frac{i}{\omega}Ak^2 = 0, \tag{40}$$

reproducing the dispersion relation of KT.

4. Derivation of the evolutionary equation

Now we take that the influence of the heating/cooling function on the wave propagation is small, in other words, we suppose that in equation (33) the third term on the left handside is much less than the other linear terms. Also, the quadratically nonlinear terms are taken into account. In the weakly nonlinear, weakly non-adiabatic case, the magnetosonic waves propagate with the phase speed *about* the speeds given by expressions (37). The weak nonlinearity and non-adiabaticity lead to *slow* evolution of the waves. (The term "slow" means that the typical times of the nonlinear and non-adiabatic modification are much greater than the wave period)

Following one certain magnetoacoustic wave, we change to the running frame of reference,

$$\xi = z - Ct, \quad \tau = t, \tag{41}$$

where C is either C_{fast} for the fast wave or C_{slow} for the slow wave. The variable τ means the *slow* time, which corresponds to the slow evolution of the wave considered due to the weak nonadiabaticity and nonlinearity. The dependence of the wave amplitude upon this slow time τ represents the difference of the wave propagation in the presence of the weak nonadiabaticity and nonlinearity from the ideal and linear case. In this frame of reference, equation (33) is rewritten as

$$-2C(2C^2-C_s^2-C_A^2)\frac{\partial^4}{\partial\tau\partial\xi^3}V_z+A\frac{C^2-C_A^2\cos^2\alpha}{C}\frac{\partial^3}{\partial\xi^3}V_z$$

$$=\frac{1}{\rho_0}\frac{\partial^3}{\partial\xi^3}\left\{-(C^2-C_A^2\cos^2\alpha)\left[CN_6+(C_s^2N_8+\frac{1}{\rho_0}N_7)\right]\right.$$

$$\left.+\frac{B_0\sin\alpha}{4\pi}C\left(-CN_1+\frac{B_0\cos\alpha}{\rho_0}N_4\right)\right\}, \tag{42}$$

where expressions for the nonlinear terms are following:

$$N_1 = -\frac{B_0C\sin\alpha}{C^2-C_A^2\cos^2\alpha}\frac{\partial V_z^2}{\partial\xi}, \tag{43}$$

$$N_4 = 0, \tag{44}$$

$$N_6 = -\frac{B_0^2}{4\pi}\frac{C^2\sin^2\alpha}{(C^2-C_A^2\cos^2\alpha)^2}V_z\frac{\partial V_z}{\partial\xi}, \tag{45}$$

$$N_7 = -\frac{\gamma(\gamma-1)\rho_0p_0}{C}V_z\frac{\partial V_z}{\partial\xi}, \tag{46}$$

$$N_8 = -\frac{2\rho_0}{C}V_z\frac{\partial V_z}{\partial\xi}. \tag{47}$$

Only first order derivatives with respect to the slow time τ are kept on the left hand side of equation (42) and neglected in the non-adiabatic and

nonlinear terms. We used expressions (28)-(31) rewritten in the moving frame of reference (41),

$$\rho = \frac{\rho_0}{C} V_z, \quad p = \rho_0 \frac{C_s^2}{C} V_z, \quad V_x = -\cot\alpha \frac{C^2 - C_s^2}{C^2} V_z,$$

$$B_x = \frac{B_0 C \sin\alpha}{C^2 - C_A^2 \cos^2\alpha} V_z, \tag{48}$$

where only fast dependence on time has been taken into account.

Using (43)—(47), we can rewrite equation (42) as

$$\frac{\partial^4}{\partial\tau\partial\xi^3} V_z + \mu \frac{\partial^3}{\partial\xi^3} V_z + \varepsilon \frac{\partial^3}{\partial\xi^3} V_z \frac{\partial V_z}{\partial\xi} = 0, \tag{49}$$

where μ and ε are the coefficients,

$$\mu = -A \frac{C^2 - C_A^2 \cos^2\alpha}{2C^2(2C^2 - C_s^2 - C_A^2)}, \tag{50}$$

$$\varepsilon = \frac{3C_A^2 C^4 \sin^2\alpha + C_s^2(\gamma+1)(C^2 - C_A^2\cos^2\alpha)^2}{2C^2(C^2 - C_A^2\cos^2\alpha)(2C^2 - C_s^2 - C_A^2)}. \tag{51}$$

Integrating equation (49), and assuming that the arbitrary constants of the integration are zero, we obtain an evolutionary equation describing the weakly nonlinear dynamics of magnetoacoustic waves in the magnetised plasma in the presence of the heating/cooling function,

$$\frac{\partial}{\partial\tau} V_z + \mu V_z + \varepsilon V_z \frac{\partial V_z}{\partial\xi} = 0. \tag{52}$$

Equation (52) is an analogue of the Burgers and Korteweg - de Vries equations (the difference is connected with taking into account of the effect of nonadiabaticity instead of dissipation or dispersion leading to appearance of the terms with highest derivatives with respect to the spatial coordinate), and is a generalisation of the equation obtained in Ref. KT for acoustic waves.

We note that equation (52) describes the weakly nonlinear and nonadiabatic evolution of either the fast magnetoacoustic wave or slow magnetoacoustic wave, but not their nonlinear coupling. The nonlinear interaction of the magnetoacoustic waves is neglected due to the single wave approximation applied above. In this approximation, it is supposed that there initially is only a wave of one certain type in the system, while another wave is absent. Nonlinear coupling of the MHD waves in the thermal unstable medium will be considered elsewhere.

Consider dependence of the coefficients μ and ϵ of evolutionary equation (52) on the problem parameters. Actually, there are two independent parameters in the problem considered, the angle of the wave propagation α and parameter β defined as a ratio of the kinetic and magnetic pressure in the plasma, $\beta = \frac{2}{\gamma} C_s^2 / C_A^2$.

Figures (2) and (3) show these two coefficients as a function of α for different values of β, for fast and slow waves, respectively. In Figure (2) can be seen for a given value of the angle both coefficients increase when β decreases but the opposite occurs for slow waves (see Figure (3)). When $\beta = 1.2$ both coefficients ($\mu C_s^2 |A|^{-1}$ and ϵ) are independent of angle α.

In the case $B_0 = 0$ ($C_A = 0$, $\beta \to \infty$), we have

$$C = C_{\text{fast}} = C_s; \quad \mu = -\frac{A}{2C_s^2}; \quad \varepsilon = \frac{\gamma+1}{2}. \tag{53}$$

Expressions (53) coincide with the expressions obtained in Ref. KT. All coefficients are isotropic (independent on the angle of the wave propagation).

5. Analysis of wave dynamics

Equation (52) is a partial differential equation of the first order, and can be easily solved analytically in an implicit form (KT),

$$V_z - F\left(\xi + \frac{\varepsilon}{\mu} V_z [1 - \exp(\mu\tau)]\right) \exp(-\mu\tau) = 0, \tag{54}$$

where the function $F(x)$ is a profile of the wave in the initial time, $V_z(z, 0) = F(z)$.

For convenience of the following consideration, we rewrite solution (54) in the laboratory frame

of reference,

$$V_z - F\left(z - Ct + \frac{\varepsilon}{\mu}\, V_z[1 - \exp(\mu t)]\right) \exp(-\mu t) = 0. \tag{55}$$

When the parameter of nonadiabaticity μ tends to zero, solution (55) becomes the well-known simple wave solution (Landau and Lifshitz, 1987)

$$V_z - F(z - Ct - \varepsilon V_z t) = 0 \tag{56}$$

describing steepening and breaking of a nonlinear wave in the absence of dissipation, dispersion and nonadiabaticity. Nonlinearity leads to generation of second harmonics, carrying out the upward energy shift in the wave spectrum. In a finite time, there appear infinite gradients in the wave shape, corresponding to a weak shock wave. If the parameter of nonlinearity ε is also zero (this takes place in the case of a wave with infinitely small amplitude), solution (56) shows that the wave propagate without a change of the wave shape. Different signs of the parameter ε define where, at the front or rear slope of the wave (or, more precisely, on the slope with positive or negative sign of derivative of the function describing the wave profile), the shock formation does take place.

To obtain the time and position of formation of the shock wave, or the time and coordinate of where the wave is breaking, it is convenient to use another implicit form of the solution of equation (52) which can be easily obtained from solution (55),

$$z - Ct + \frac{\varepsilon}{\mu} V_z(1 - \mathrm{e}^{\mu t}) + f(V_z \mathrm{e}^{\mu t}) = 0, \tag{57}$$

where the function f is defined implicitly by the initial shape of the wave, $z = f(V_z)$.

Following Ref. KT, we find that the breaking time is given by the expression

$$t_{breaking} = -\frac{1}{\mu} \log\left(1 + \frac{\mu}{\varepsilon} f'\right), \tag{58}$$

where f' is the value of the derivative of the wave profile at the inflection point. In the adiabatic case ($\mu = 0$), formula (58) reduces to the well-known formula for a simple wave (Landau and Lifshitz, 1987). In the unmagnetised case $B_0 = 0$, formula (58) reduces to the formula for acoustic wave breaking time. Since the parameter ε is positive for all angles of propagation for both fast and slow waves, the shock appears at the slope with $f' < 0$ for all angles and both waves. The specific value of the breaking time depends on the type of the wave considered and the angle of propagation.

Modification of the wave evolution, carried by finite nonadiabaticity, depends on the sign of the parameter μ. A positive μ acts similarly to dissipation, leading to wave decay. In such a case, equation (58) defines a maximum value of $\mu = \mu_{\max} = \varepsilon/|f'|$ beyond which the damping of the disturbance by non-isentropic processes occurs in time shorter than the time at which nonlinear effects go into play, and shocks waves can not be formed. A negative μ corresponds to the case of thermal instability, when the wave amplitude grows in time. (More strictly, in this case, this effect should be called as a thermal overstability or oscillatory instability). According to (50) and Fig. 2a and 3a, the sign of the coefficient of nonadiabaticity is opposite to the sign of the parameter A introduced in (35). Consequently, the amplification of the finite amplitude magnetoacoustic wave takes place when the parameter A is positive.

Different regimes of evolution of the initial Gaussian pulse,

$$V_z(z) = V_0 \exp(-z^2/L^2), \tag{59}$$

where V_0 is an amplitude and L is a characteristic length of the pulse in the initial time $t = 0$, are shown on Figure 4. Independently on the sign of the parameter μ, the leading slope of the pulse is steepening due to nonlinearity. In the case of the active medium, $\mu < 0$, the amplitude of the pulse is amplified. In the dissipative medium, $\mu > 0$, the amplitude decays. The sketch shown on Fig. 4 is valid qualitatively for both slow and fast waves. The quantitative dif-

ferences arise from different angular dependences of the speed of propagation and nonlinear and nonadiabatic coefficients (see Fig. 1, 2 and 3).

6. Astrophysical Applications

Now, we shall apply the theory developed above to two special astrophysical plasma systems, a high temperature atomic ($T > 10^4$ K) plasma and the cold ($T < 10^3$ K) molecular ISM gas.

6.1. High Temperature Gas

An optically thin plasma in the range of temperature $10^4 < T < 10^7$ K radiates according to the relation

$$\Lambda = \rho^2 \Lambda_i (\frac{T}{T_i})^\eta, \quad (erg\ cm^{-3}\ s^{-1}) \ , \qquad (60)$$

where the coefficients Λ_i and the exponent η are well known in the above range of temperature (Vesecky et. al. 1979). Therefore, if one assumes that the plasma heats as $\sim \rho^a T^b$ (erg cm^{-3} s^{-1}) (Dahlburg & Mariska 1988), then, it becomes isentropically unstable ($\mu < 0$) in the ranges of temperature $1.56 \times 10^4 \leq T < 3.16 \times 10^4$ K, $2.51 \times 10^5 \leq T < 6.31 \times 10^5$ K and $2.0 \times 10^6 \leq T < 3.16 \times 10^7$ K for $2.51 \times 10^5 \leq T < 6.31 \times 10^5$ K for heating by coronal current dissipation ($a = b = 1$) and heating by Alfvén mode/mode conversion ($a = b = 7/6$), and in the range $2.51 \times 10^5 \leq T < 6.31 \times 10^5$ K for a constant heating per unit of mass ($a = 1$ and $b = 0$) . Thus, in the Sun, the enhancement of shock waves formation due to thermal effects will be relevant in the chromosphere, transition region and corona where the corresponding e-folding time $t_e = 2C_s^2|A|^{-1}$, for thermal instability is of the order of 5, $2 \times 10^2$and 6×10^5 s, respectively. For disturbances with $L/V_0 \approx 1/|\mu|$, the breaking times are of the order of $0.626|\mu|^{-1}$ for a magnetic field of about 1 G . This value decreases when the intensity of the magnetic field increases.

On the other hand, Interstellar Medium regions with temperature in the range $2.51 \times 10^5 \leq T < 6.31 \times 10^5$ K (coronal gas) will be isentropically unstable with e-folding times of the order of 3×10^{15} s and a breaking time $t_b \approx 0.609|\mu|^{-1}$ for disturbances with $L/V_0 \approx 1/|\mu|$, if one assumes a constant heating per unit of mass ($a = 1$ and $b = 0$) and a magnetic field B_G of about $2 \times 10^{-6} G$.

6.2. Cold Molecular Gas

For diffuse and cold ($T < 10^3$ K) ISM molecular gas (Van Dishoeck & Black, 1986; Viala, 1986) one may assume, as a first approximation, a heating by cosmic rays and grain photoelectrons (Ibañez and Parravano 1994 and references therein), and a cooling (Hollenbach & Mc Kee, 1979; Hollenbach,1988) functions to be of the form

$$\Gamma = H_0 \rho \ , \quad (erg\ cm^{-3}\ s^{-1}) \ , \qquad (61)$$

$$\begin{aligned} \Lambda &= 8.5 \times 10^{-5} n^2 L(CO) + n_{H_2}^2 L(H_2) + \\ & \quad n_H n_{H_2} L(H) \ , \quad (erg\ cm^{-3}\ s^{-1}) \ , \end{aligned} \qquad (62)$$

where n_H, $n_{H_2}$and n are the number density of the atomic, molecular hydrogen, and the total number density of particles, respectively. $L(CO)$, $L(H_2)$ and $L(H)$ are the cooling efficiencies of the CO and H_2 molecules and atomic hydrogen, respectively. In denser molecular regions other heating processes becomes dominant with more complex dependence of Γ on ρ and T, detailed study of which will be reported elsewhere.

The range of temperature (and particle density) at which the gas is isentropically unstable ($A < 0$), is strongly dependent on the ratio n_H/n and on the energy input parameter H_0. In particular, for $n_H/n = 0.9$ these ranges are $115 \lesssim T \lesssim 1905$ K ($136 \gtrsim n$) and $90 \lesssim T \lesssim 1865$ K ($3 \gtrsim n$) for $H_0 = 10^{-2}$ and $H_0 = 10^{-4}$, respectively. However, it is found that the above thermal instability occurs for high values of n_H/n, i.e.

for low concentrations of H_2 and therefore, when the transition from atomic to molecular gas takes place. The range of temperature for which the gas is thermally unstable decreases when n_{H_2}/n increases and is quenched for high enough values of n_{H_2}/n ($\gtrsim 0.13$).

The ratios between the breaking and the e-folding time, t_b/t_e for a disturbance with Gaussian profile and $L/V_0 = 10^{16}\ s$, have been plotted in Figures 5a and 5b, in absence of a magnetic field (dash-dot line) as well as for the Galactic field value $B_G = 2 \times 10^{-6}\ G$, for three different values of the angle α, for the fast (a) and the slow (b) mode . The breaking times show maxima at a temperature of about 125 K, but the breaking times for the magnetoacoustic modes are shorter than that corresponding to the hydrodynamic mode. It means that the effect of the magnetic field is to decrease the time when the breaking of the wave occurs. For the fast mode, the effect of increasing the angle between the wave number and **B** is to increase the corresponding breaking time . This dependence reverses for the slow mode (see Figs. 5a and 5b).

Figure 5c shows the dependence of t_b/t_e on H_0 for the fast (dash-line), slow (continuous line) and hydrodynamic (dot-line) mode. The gross effect of increasing H_0 is to increase the corresponding breaking times.

The distance between the point where the disturbance is originated to the point where the shock is formed, is of the order of $C_s t_b$. Therefore, from above results one may conclude that strong inhomogeneities are expected to be formed at time scales and scale-lengths shorter than those for which the molecular gas becomes thermally unstable.

7. Discussion

We considered dynamics of magnetohydrodynamic waves in a nonadiabatic plasma taking into account effects of finite wave amplitude. The nonadiabaticity was modelled by a general cooling/heating function of thermodynamic parameters of the plasma. Depending on a general structure of the nonadiabatic function, there are two different regimes of the wave evolution in the system, either amplification or decay. The nonlinearity, through generation of highest harmonics, leads to the wave steepening and, therefore, to formation of shock waves. Our analysis was restricted by this shock formation time. In the linearly unstable case (usually called the thermal instability), the formation of the shock takes place quicker than in adiabatic case. Consequently, nonlinearity depresses the thermal instability of magnetoacoustic waves, whose growth is accompanied by the wave steepening and lasts until the shock has been formed.

In the single wave approximation, we have derived an evolutionary equation describing development of weakly nonlinear magnetoacoustic waves in a weakly nonadiabatic medium. This equation is valid for one magnetoacoustic wave (either slow or fast) providing that another wave is absent from the system. Of course, in realistic situation, both slow and fast waves are excited simultaneously. Nevertheless, the difference in their propagation speeds gives us a possibility to restrict our consideration by the single wave approximation, because the waves occupy different locations in time after the excitation (they are spatially de-tuned from each other). Since the magnetic field brings spatial anisotropy in the system considered, parameters of the magnetoacoustic wave propagation (the speed, the rate of growth or decay, the breaking time) depend upon the direction of the wave vector. The third magnetoacoustic mode, the Alfvén wave, is not subject to the influence of the nonadiabaticity (as far as the quadratic nonlinear terms approximation holds). This does not exclude a possibility of the indirect effect of the nonadiabaticity of the plasma on the Alfvén wave through nonlinear coupling with the magnetoacoustic wave amplified by thermal instability. The nonlinear excitation of the Alfvén waves is described by

Eq. (32). According to above discussion, the active non-adiabaticity of the plasma leads to excitation of magnetoacoustic waves perturbing the variables B_x, V_x, V_z, ρ and p. Expressing all variables through V_z by (28)-(31), we can re-write the nonlinear Alfvén wave equation (32) for two variables, V_y and V_z:

$$\left(\frac{\partial^2}{\partial t^2} - C_{Az}^2 \frac{\partial^2}{\partial z^2}\right) V_y =$$

$$\frac{\partial V_z}{\partial z}\frac{\partial V_y}{\partial t} - \frac{\partial}{\partial t}\left(V_z \frac{\partial V_y}{\partial z}\right) - C_A^2 \frac{\partial^2}{\partial z^2}\left(V_z D_t^{-1} \frac{\partial V_y}{\partial z}\right), \tag{63}$$

where D_t^{-1} is an inverse differential operator. In the case of a harmonical magnetoacoustic wave, Eq. (63) describes generation of the Alfvén wave by a parametric instability. But, in the case considered, the function V_z is prescribed by expression (54). The spectrum of the exciting magnetoacoustic wave is much reacher than harmonical and evolves in time and space. However, qualitatively, we can expect that Alfvén waves are nonlinearly generated by the thermal instability through the discussed mechanism. This phenomenon has been omitted from this consideration.

The theory developed here may be applied, in particular, to an interpretation of origin of density, temperature and magnetic field variations in astrophysical plasmas, which can be associated with the magnetoacoustic weak shock waves (see, Krasnobaev, 1975). With respect to the previously analysed case of the unmagnetised plasma, the presence of the magnetic field gives several advantages for the observational registration of these phenomena. Firstly, in the vicinity of the shock front, the sharp inhomogeneity of the absolute value of the magnetic field takes place, which can be directly registered by analysis of variation of parameters of gyro-emission and Zeeman splitting. Even if the wave amplitude is sufficiently small, the sharp gradients in the absolute value of the magnetic field in the vicinity of the shock are accompanied by high densities of the electric currents (the transversal component of the current density is proportional to the spatial derivative of the magnetic field perturbation, $j_y = -(c/4\pi)\partial B_x/\partial z$). These current concentrations (or current sheets, in the ideal limit) can be regions where particle acceleration takes place. Secondly, slow and fast magnetoacoustic perturbations, initially excited somehow, propagate with different speeds and growth rates and, therefore, form different spatial structures at different distance from their source. According to formula (48), perturbations of the magnetic field are either positively or negatively correlated with perturbations of the plasma density, depending on the type of the mode (slow or fast), which gives us a possibility to distinguish fast and slow modes in analysed data.

From the point of view of the nonlinear wave dynamics, propagation of the MHD waves in the optically thin plasma is a very attractive problem, because the plasma is acting in this case as an active medium, supplying energy to the waves. The simplest model considered here can be further developed. For example, high frequency dissipation (modelled by introduction of a Burgers term in equation (52)), will compete with the amplification and depress nonlinear generation of highest harmonics. When these phenomena are in balance, there exist dissipative stationary waves with "saw-teeth" shape. Also, the generation of the second harmonics and so the energy transfer to small scales can be depressed by nonlinear dispersion (Whitham, 1974). This process is based upon either cubic nonlinearity, or quadratic nonlinearity through cascade self-interaction. The last mechanism has two stages: nonlinear generation of second harmonics and interaction of the second harmonics with a backward wave. In the case of magnetoacoustic waves, both mechanisms are acting together (Nakariakov et al., 1997b) leading to the appearance of a cubically nonlinear term in the evolutionary equation. Obviously, the cubically nonlinear effects should be taken into account only

when quadratically nonlinear processes are suppressed due to, e.g., linear high frequency dispersion. Effects of dispersion (connected possibly with electron inertia or, which is more important for astrophysical applications, with inhomogeneity of the plasma) can be modelled by introduction of a Korteweg - de Vries (or Benjamin - Ono or Leibovich - Roberts, depending on the specific situation) term in evolutionary equation (52). Particularly, the dispersion of the medium can give a possibility for existence of autosolitons and stochastic dynamical regimes (see, e.g. Engelbrecht, 1989, Nakariakov and Roberts 1999). These phenomena can be registered in astrophysical plasmas and be applied to interpretation of the observations.

Finally, we must remark that in atomic as well as in molecular plasma occurring in astrophysics, many other physical effects likely go into play, such as inhomogeneity in the background due to self-gravitation and/or to radiation transport, and dissipative effects (viscosity, thermal conduction, ionization-recombinations and chemical reactions). These phenomena have to be taken into account in more developed models. However, the above results indicate that inside thermally unstable plasma regions, substructures can be originated by nonlinear effects in times shorter than an e-folding time for evolving the isentropic thermal instability.

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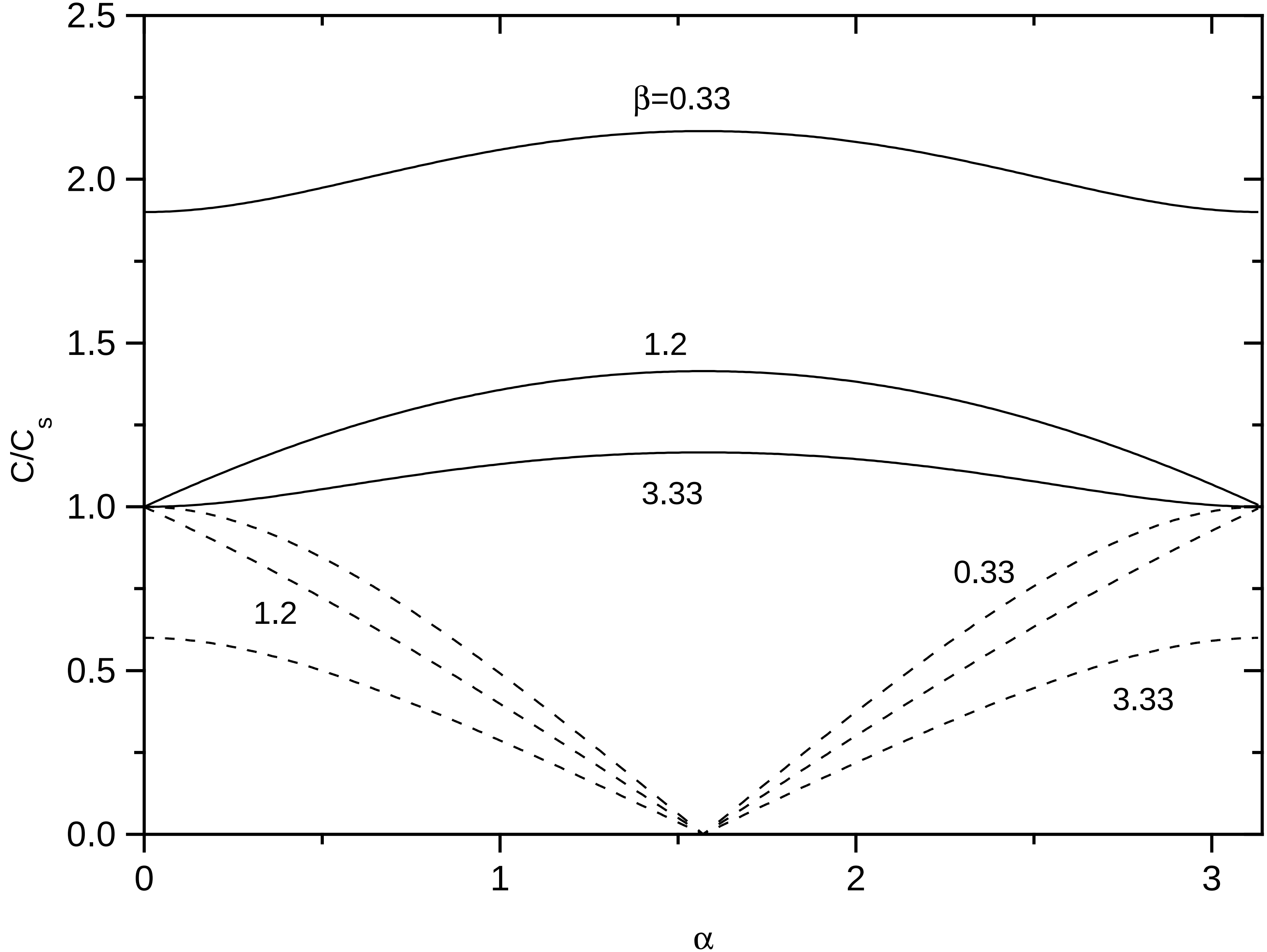
Fig. 1.— The linear speeds C/C_s as a function of α for different values of β. Continuous and dashed lines correspond to the fast and slow modes, respectively.

Fig. 2.— (a) The non-adiabaticity $(\mu C_s^2 |A|^{-1})$ and (b) the nonlinearity ϵ parameters as functions of α for different values of β corresponding to the fast wave.

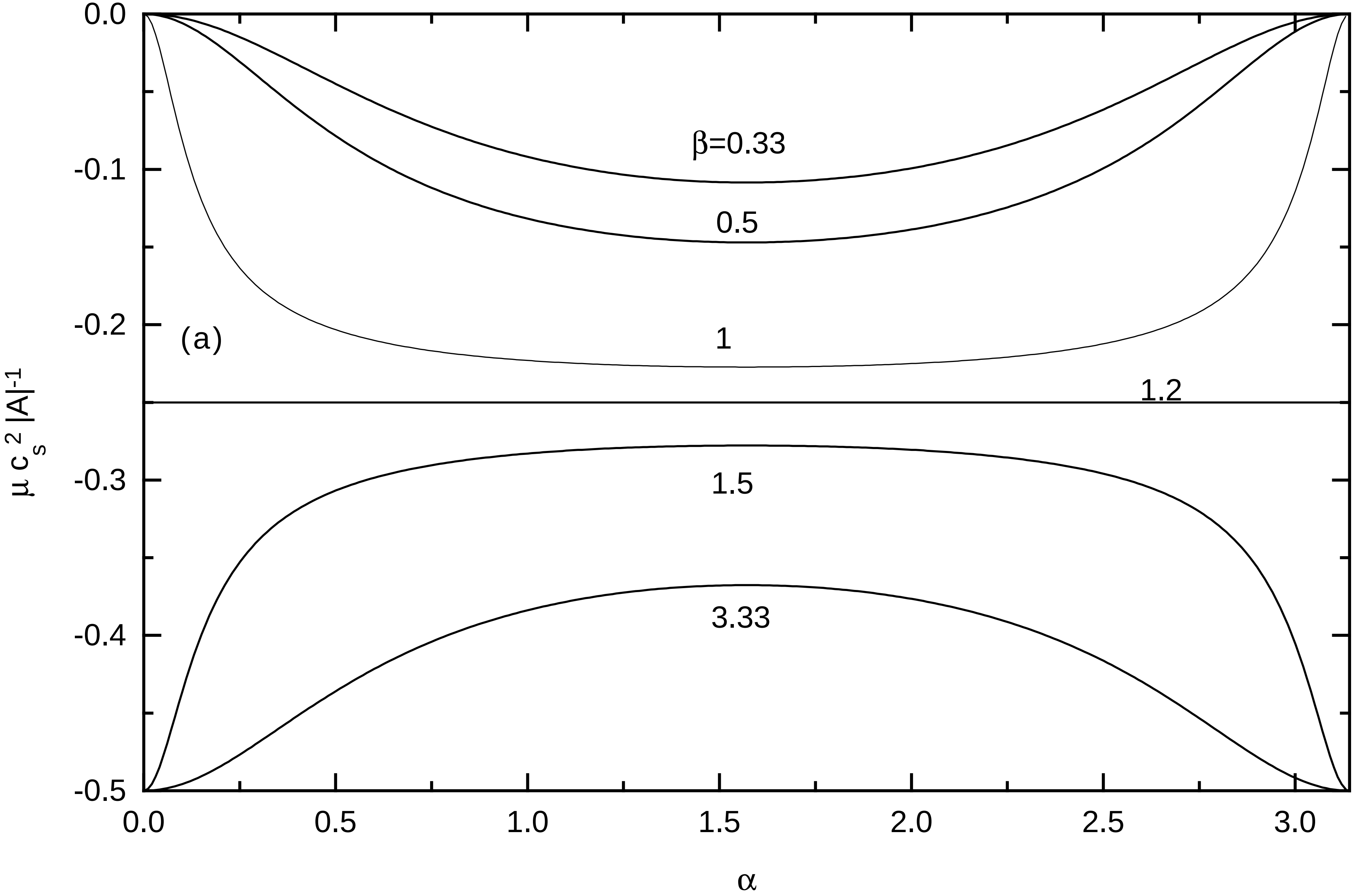
Fig. 3.— As Figure 2 for the slow wave.

Fig. 4.— Development of the shape of the initial Gaussian magnetoacoustic pulse with the width L situated initially $t = 0$ in $z = 0$, and the indicated time in units of L/V_0. In the active medium (a) ($\mu < 0$) the pulse is amplified. In the dissipative medium (b) $\mu > 0$ the pulse decays.

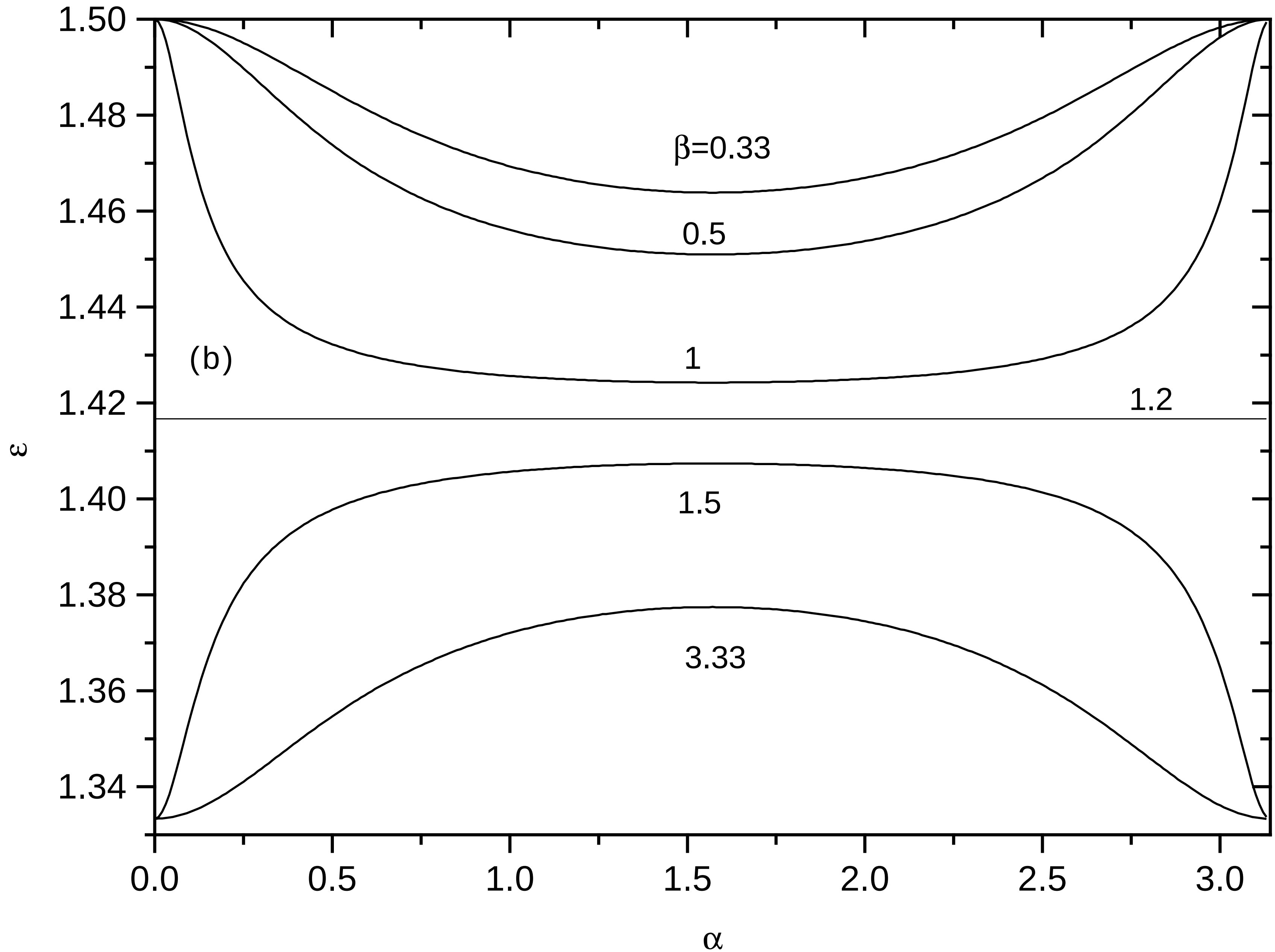
Fig. 5.— The breaking times for the fast (a) and slow mode (b) as a function of temperature T, for $H_0 = 10^{-3}$, $B = 0$ (dash-dot line), $B = B_G$, $n_H/n = 0.9$ and three values of the angle $\alpha = \pi/36$ (dash line), $\pi/4$ (solid line) and $17\pi/36$ (dot line). Fig.5c is as Fig. 5a for two different values of H_0, 10^{-3} and 10^{-4}.



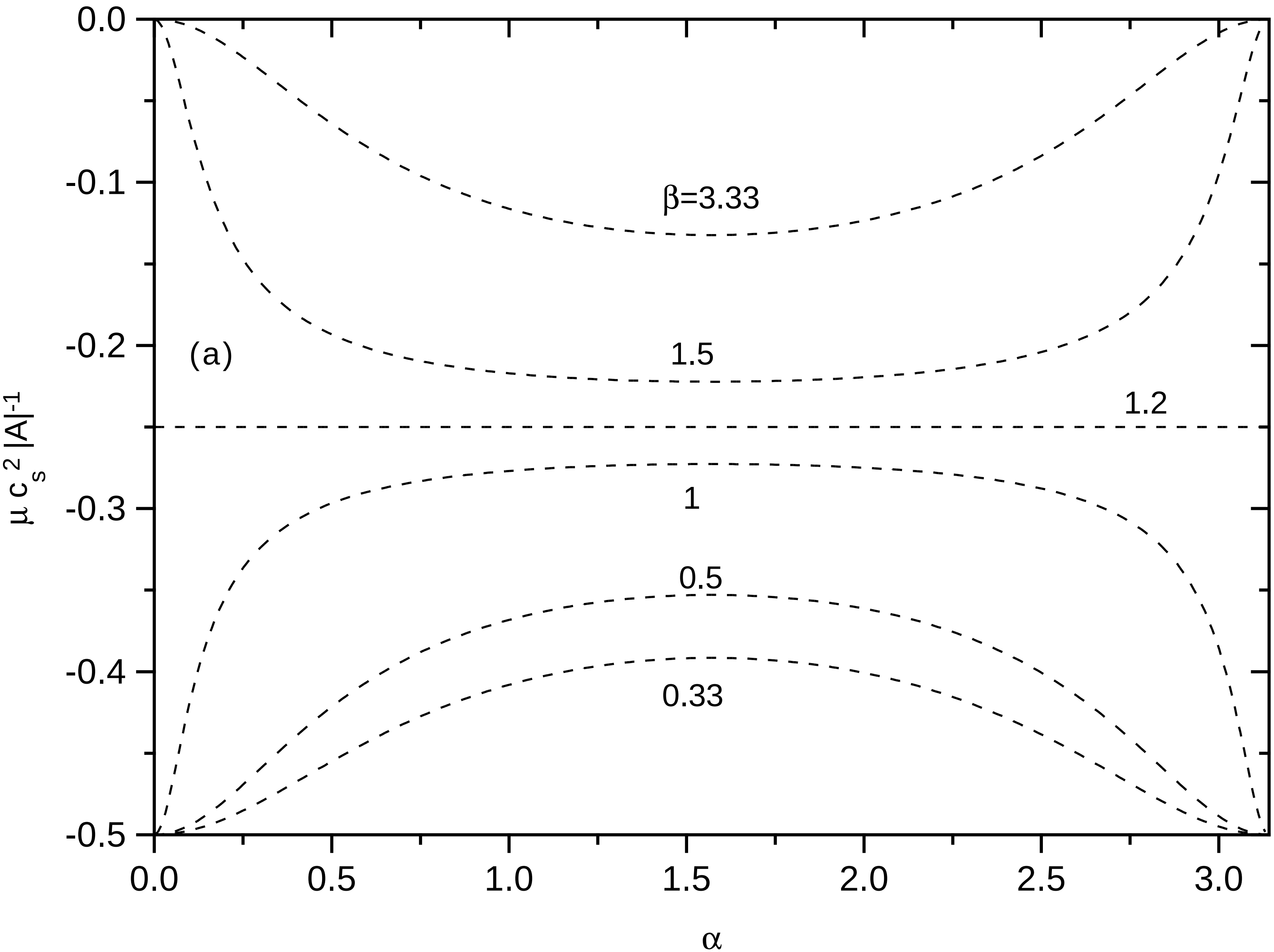
β=0.33
1.2
3.33
1.2
0.33
3.33
C/C_s
α
0.0
0.5
1.0
1.5
2.0
2.5
0
1
2
3



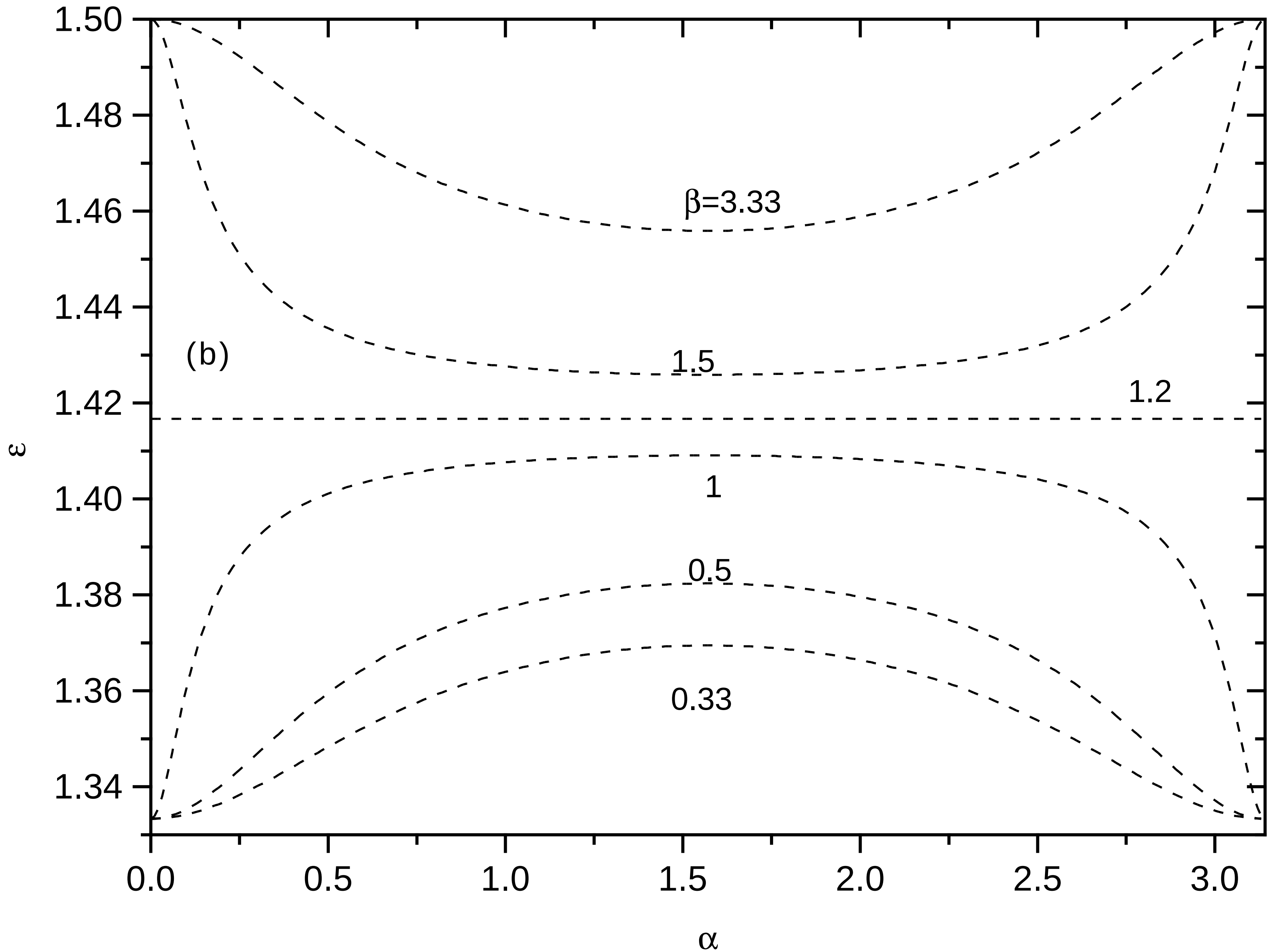
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
μ c s 2 |A|-1
β=0.33
0.5
1
1.2
1.5
3.33
(a)
0.0
0.5
1.0
1.5
2.0
2.5
3.0
α



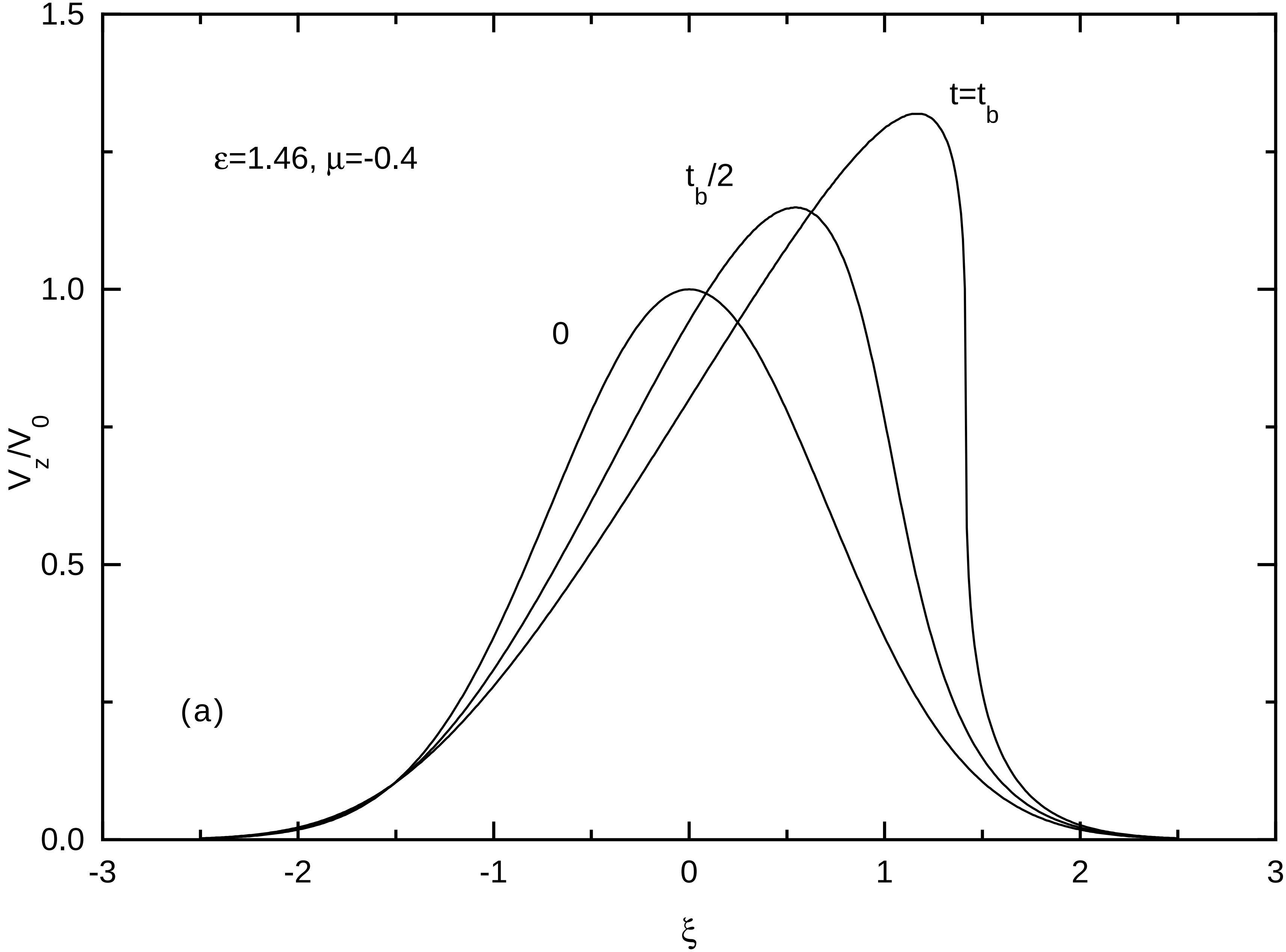
1.50
1.48
1.46
1.44
1.42
1.40
1.38
1.36
1.34
ε
0.0
0.5
1.0
1.5
2.0
2.5
3.0
α
β=0.33
0.5
1
1.2
1.5
3.33
(b)



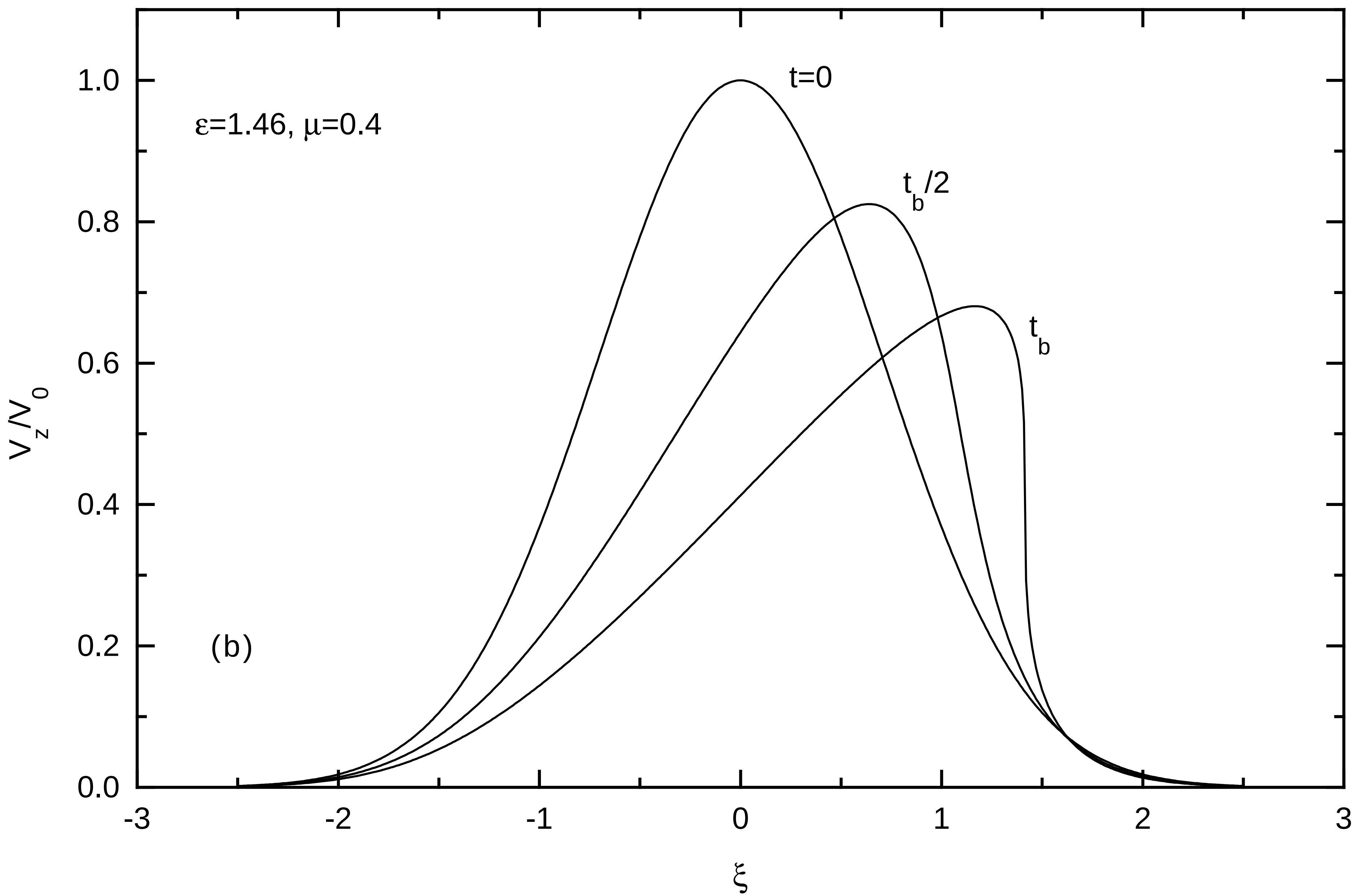
0.0
-0.1
-0.2
-0.3
-0.4
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\mu c_s^2 |A|^{-1}$
α
(a)
β=3.33
1.5
1.2
1
0.5
0.33



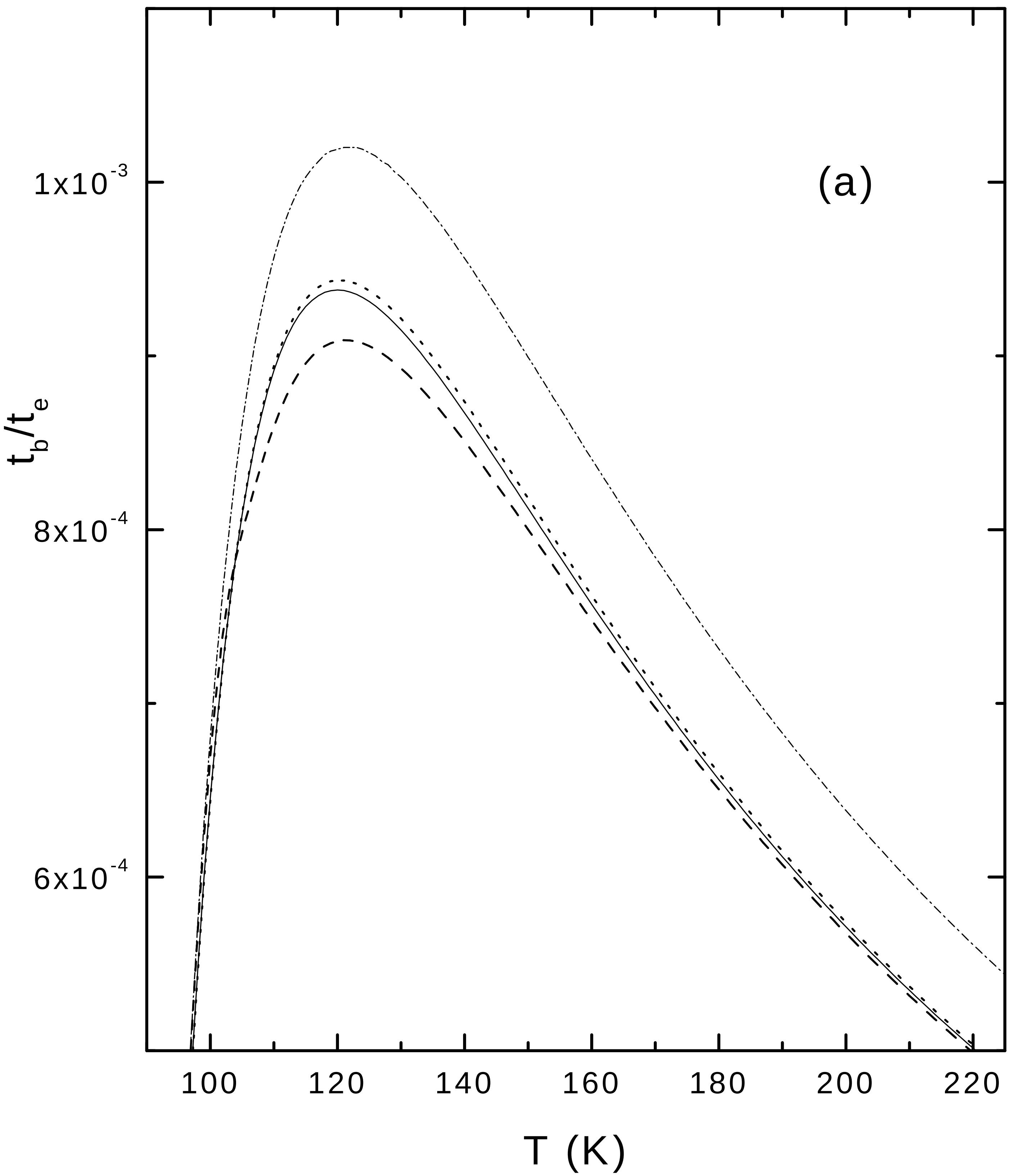
1.50
1.48
1.46
1.44
1.42
1.40
1.38
1.36
1.34
ε
0.0
0.5
1.0
1.5
2.0
2.5
3.0
α
(b)
β=3.33
1.5
1.2
1
0.5
0.33



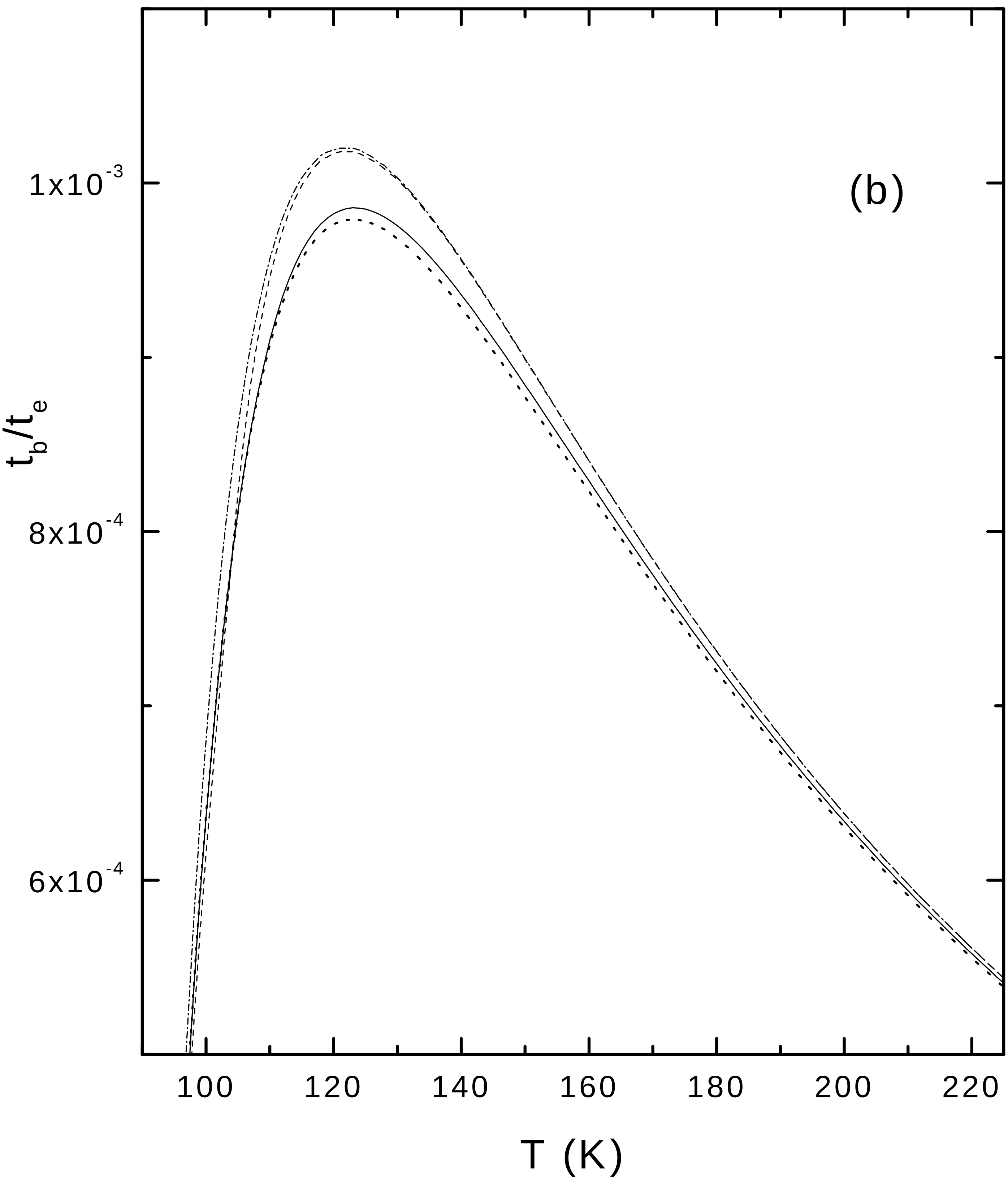
1.5
1.0
0.5
0.0
-3
-2
-1
0
1
2
3
ξ
V_z/V_0
ε=1.46, μ=-0.4
t=t_b
t_b/2
0
(a)



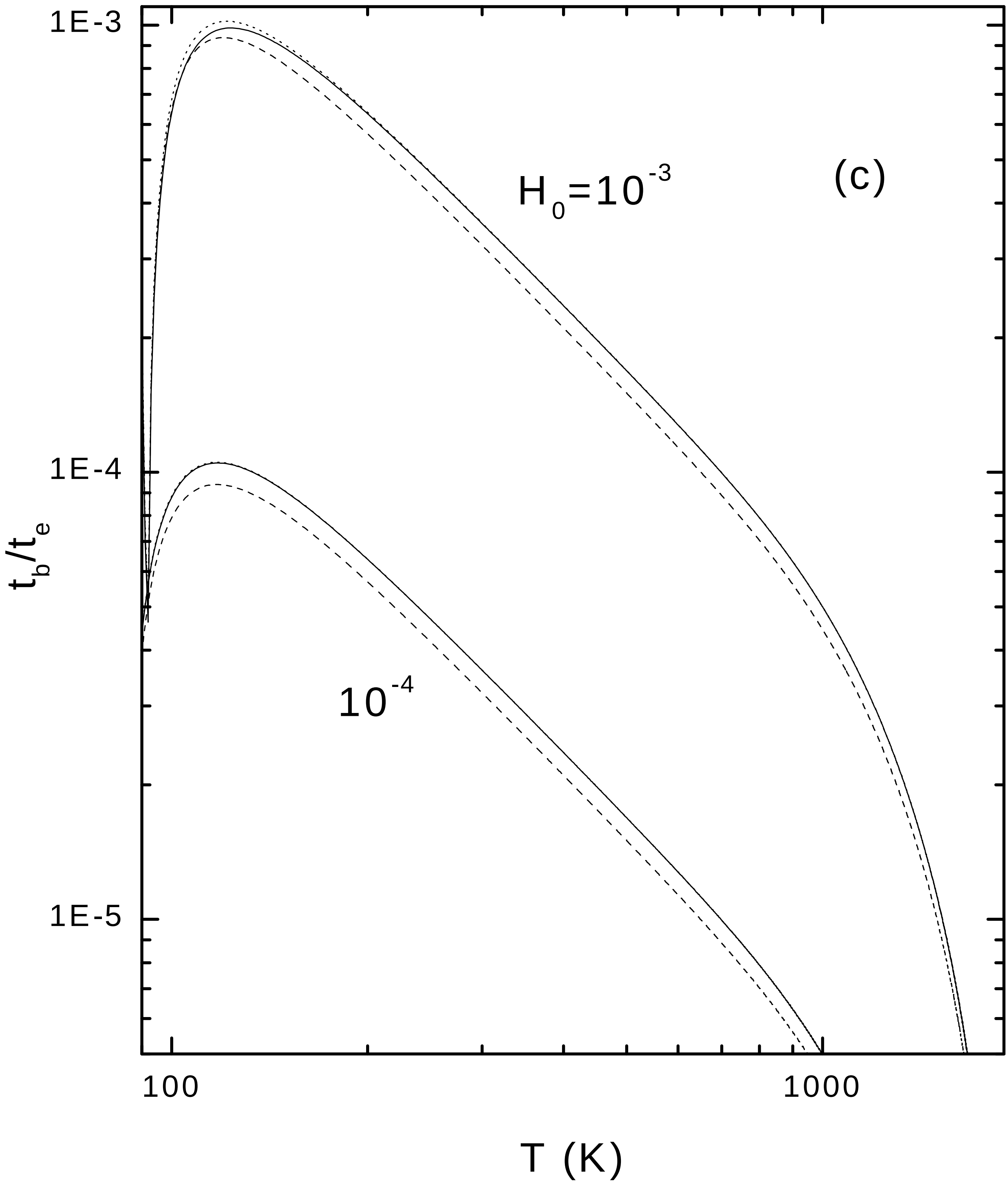
t=0
t_b/2
t_b
ε=1.46, μ=0.4
(b)
V_z/V_0
ξ
1.0
0.8
0.6
0.4
0.2
0.0
-3
-2
-1
0
1
2
3



(a)
$1x10^{-3}$
$8x10^{-4}$
$6x10^{-4}$
t_b/t_e
100
120
140
160
180
200
220
T (K)



(b)
1x10^-3
8x10^-4
6x10^-4
t_b/t_e
100
120
140
160
180
200
220
T (K)



1E-3
1E-4
1E-5
100
1000
T (K)
t_b/t_e
$H_0=10^{-3}$
10^{-4}
(c)